


Problem Solving by
Case-Based Reasoning
 PART 1


Machine Learning
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Agenda


1. Introduction to CBR **PART 1**
2. Knowledge and Case Representation
3. Similarity
4. Similarity-Based Retrieval
5. Solution Adaptation **PART 2**
6. Learning in Case-Based Reasoning
7. Applications
8. References



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1. INTRODUCTION

What is Case-Based Reasoning?




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Case-Based Reasoning is ...

- an approach to model the way humans think
- an approach to build intelligent systems

Central Ideas:


- store experiences made → as **cases**
- solving a new problem do the following
 - recall **similar** experiences (made in the past) from memory
 - reuse that experience in the context of the new situation (reuse it partially, completely or modified)
 - new experience obtained this way is stored to memory again



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Classification of CBR

- sub-discipline of Artificial Intelligence
- belongs to Machine Learning methods
 - learning process is based on analogy (not on deduction or induction)
 - best classified as supervised learning
- one of the few commercially/industrially really successful AI methods
 - customer support, help-desk systems: diagnosis and therapy of customer's problems, medical diagnosis
 - product recommendation and configuration: e-commerce
 - textual CBR: text classification, judicial applications (in particular in the countries where common law (not civil law) is applied (like USA, UK, India, Australia, many others))
- applicability also in ill-structured and bad understood application domains




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Case-Based Reasoning and Cases

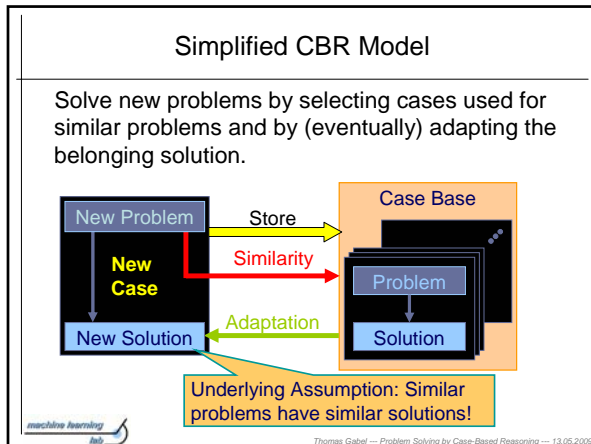
- „Case-Based Reasoning is [...] reasoning by remembering.“ Leake, 1996
- „A case-based reasoner solves new problems by adapting solutions that were used to solve old problems.“ Riesbeck & Schank, 1989
- „Case-Based Reasoning is a recent approach to problem solving and learning [...]“ Aamodt & Plaza, 1994
- „Case-Based Reasoning is both [...] the ways people use cases to solve problems and the ways we can make machines use them.“ Kolodner, 1993

What is a Case?

- a) Cognitive Science View:
Cases are abstractions of events, that are limited in space and time; they represent episodic knowledge.
- b) Technical View:
A case is a description of a problem situation (that actually occurred) together with certain experiences that could be obtained during processing and solving the problem.



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- ### When is CBR of Relevance?
- When a domain theory does not exist, **but example cases are easy to find**.
 - When an expert in the domain is not available, is too expensive, or is incapable of articulating verbally his performance, **but example cases are easy to find**.
 - When it is difficult to specify domain rules, **but example cases are easy to find**.
 - When cases with similar solutions have similar problem descriptions.
 - i.e. there exists a similarity metric for problem descriptions and a corresponding set of adaptation rules
 - When a case base already exists.
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- ### Contents of a Case
- | | |
|---|---|
| <p>Mandatory</p> <ul style="list-style-type: none"> • problem part • solution part | <p>Optionally</p> <ul style="list-style-type: none"> • context (e.g. justifications) • pointer to other relevant cases • solution quality assessment • steps of the solution |
|---|---|
- The main difficulty arises, when the actual situation is **not identical** to the previous one. Then, **inexactness** is involved.
- A main feature of CBR techniques is that they allow inexact / approximate reasoning in a controlled manner.
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- ### A Simple Example Scenario: Call Center (I)
- **Task: Technical Diagnosis of Car Faults**
 - symptoms are observed (e.g. engine does not start) and values are measured (e.g. battery voltage = 6.3V)
 - goal: find the cause for the failure (e.g. battery empty) and a repair strategy (e.g. change battery)
 - **Case-Based Diagnosis**
 - a case describes a diagnostic situation and contains
 - a description of the symptoms
 - description of the failure and the cause
 - description of a repair strategy
 - store a collection of cases in the case base
 - find a case similar to the current problem and reuse the repair strategy
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A Simple Example Scenario: An Example Case (II)

- A case describes a particular diagnostic situation.
- A case records several features and their specific values occurred in that situation.
 - A case is not a general rule.

Case 1	Problem (Symptoms):	Attributes (Features)	Values
	- Problem : front light does not work - Car : VW Golf IV, 1.6l - Year : 1998 - Battery Voltage : 13.6V - State of Lights : ok - State of Light Switch : ok	String Symbolic Data Type (Polo, Golf,...) Numeric Data Type [0:50]=R	
	Solution:		
	- Diagnosis : front light fuse defect - Repair : repair front light fuse		

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A Simple Example Scenario: Solving a New Diagnostic Problem (III)

Case Base with Two Cases

- each case describes one particular situation
- all cases are independent of one another

A New Problem (Query) Has to Be Solved

- we make several observations in the current situation
- observations define a new problem
- not all attribute values have to be known
- Note: The new problem is a "case" without solution part

Case 1	Problem (Symptoms): - Problem : front light does not work - Car : VW Golf IV, 1.6l - Year : 1998 - Battery Voltage : 13.6V - State of Light : ok - State of Light Switch : ok	Solution: - Diagnosis : front light fuse defect - Repair : repair front light fuse
Case 2	Problem (Symptoms): - Problem : front light does not work - Car : Audi A4 - Year : 2002 - Battery Voltage : 12.9V - State of Light : surface damaged - State of Light Switch : ok	Solution: - Diagnosis : bulb defect - Repair : replace front light

Compare the new problem with each case and select the most similar one!

→ **CASE 1**

Questions:

- When are two cases similar?
- How to rank the cases according to their similarity?
- How to reuse the solution of the corresponding case?

Note: Similarity is the most important concept in CBR. Similarity may be assessed based on the similarity of each feature, while the importance of different features may vary (feature weighting).

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A Simple Example Scenario: Reuse and Retain (IV)

- Reuse**
 - adapt the solution
 - how do differences in the problem affect the solution
- Retain**
 - if diagnosis is correct: store new case
 - add case to case base

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CBR Cycle (R4, [Aamodt&Plaza, 1994])

- retrieve:** find most similar case(s)
 - similarity measures
 - explanation-based methods
 - case-base organisation (data structures)
- reuse:** transform/adapt solution
 - different types of solution transformation (none, interactive, derivational, etc.)
 - different methods (rule-based, constraint satisfaction, model-based etc.)
- revise:** verify/improve solution
 - no verification
 - verification by simulation
 - verification in the real world
- retain:** keep the experience made
 - learn new cases
 - learn similarity assessment
 - learn case base organization
 - learn solution adaptation

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Advantages of CBR (I)

- Avoidance of High Knowledge Acquisition Effort**
 - case knowledge is usually easily acquirable
 - not much general knowledge required
- Simpler Maintenance of the Knowledge in the System**
 - maintenance by adding/removing cases from the case base
 - cases are independent of one another and easily interpretable (even for non-experts)
- Facilitation of Intelligent Retrieval (compared to data-base systems)**
 - DBMS often give too few/many results

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Advantages of CBR (II)

- High Quality of Solutions for Poorly Understood Domains**
 - case-based systems can be made to retain only "good" experience in memory
 - if only little adaptation is necessary for reuse, this will not impair the solution's quality too much
- High User Acceptance**
 - provided solution corresponds to actual experience → may increase trust in the solution
 - selected case and solution adaptation can be explicitly presented to the user
 - problems of rule-based / neural network-based systems
 - black boxes
 - inference process is not traceable or hidden
 - provided solutions are difficult to explain

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Typical Application Fields (I)

- Remarks concerning synthetic tasks:**
 - main focus is on composing a complex solution from simpler components → focus is often on solution adaptation
 - configuration: e-commerce scenario → product configuration (e.g. personal computers)
 - design: reuse of construction plans in civil engineering
 - planning: production planning

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Typical Application Fields (II)

- Remarks concerning analytical tasks:**
 - main focus is on analysing a given situation
 - classification** (assign objects to a class $K_i \in \{K_1, \dots, K_n\}$) → e.g. recognition of sponges
 - diagnosis** (classification + verification + therapy) → e.g. fault diagnosis in Airbus engines
 - evaluation/regression** (like classification, but assignment of real-valued assessments): → e.g. credit risk assessment
 - decision support** (search for specific information relevant for decision-making) → e.g. web-based product catalogues, like online travel agencies
 - prediction** (like classification + time dependency) → e.g. prediction of the probability of failure of a machine's part

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CBR for Classification

- A classifier for a set M is a mapping $f: M \rightarrow I$ (where I is a finite index set).
 - A case-based classifier is given by a case base, a similarity measure and the principle of the nearest neighbour.
- Definition:** Given a case base CB , a similarity measure sim and an object (problem) $q \in M$, we call $c=(p,s) \in CB$ the **Nearest Neighbour** to q , if: for all $(p',s') \in CB$ it holds $sim(q,p) \geq sim(q,p')$.
- NN-Classification: Each new object (query) $q \in M$ is assigned the class $s \in I$ of q 's nearest neighbour in CB .
- Note: The classifier defined by the pair (CB, sim) is not unique, if there is more than one nearest neighbour.
- Extension to kNN-Classification: The k most similar neighbours of q are considered. Typically, a majority voting is applied to determine the class of the query q .

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k-Nearest Neighbor Classification

- Demonstration video on the k-NN classifier



(c) Antal van den Bosch,
Tilburg University

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2. KNOWLEDGE AND CASE REPRESENTATION

*What forms of knowledge are parts of a CBR system?
How can cases be represented?*

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Knowledge Container Model [Richter, 1989]

- „In order to solve problems, one needs knowledge.“
- Knowledge of a CBR System
 - vocabulary: knowledge representation
 - retrieval: similarity assessment (measures)
 - solution transformation: rules
 - cases
- Knowledge Management
 - as the environment may change, maintenance of the containers' contents over the lifetime of the CBR system is crucial to guarantee its continued usability



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Case Contents

Problem / Situation Information → must cover all the information that is necessary to decide if this case is applicable for a new situation <ul style="list-style-type: none"> target of the problem constraints characteristics → new situation = query	Solution → contains all the information that describes a solution to the problem sufficiently accurately <ul style="list-style-type: none"> solution itself justifications possible alternative solutions steps that were tried, but failed
	Solution Evaluation → feedback from the real world → How good was the solution for the problem?

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Case Representation Formalisms (I)

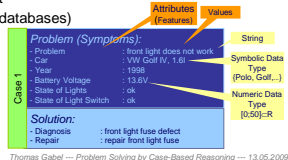
Attribute-Value Based Case Representation

- Case (problem and solution) is represented by pairs of attributed and belonging values.
 - e.g.: price = 9.95€
- Set of attributes $\{A_1, \dots, A_n\}$ is (in general) fixed for all cases.
- To each attribute A_i there is an associated domain D_i and for each attribute's value it holds $a_i \in D_i$, e.g.
 - numerical attributes (Integer or Real or subsets of those)
 - symbolic attributes (finite domains, $D_i = \{d_1, \dots, d_m\}$)
 - textual attributes (strings)
- Note:
 - Choice of attributes and corresponding domains to represent cases represents general knowledge: vocabulary knowledge.
 - Choice of domains is mainly influenced by the requirements for similarity computation and solution adaptation.

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Case Representation Formalisms (II)

- Choice of attributes
 - must allow for the decision whether a case and a new situation are similar
 - should avoid redundancies
 - should represent independent properties of a case
- Disadvantages
 - no structural or relational information is representable
 - no ordering information (e.g. sequence of actions) is representable
- Advantages
 - straightforward representation
 - easy to understand and implement
 - cases are easy to store (usage of databases)
 - efficient retrieval
- Example
 - recall the example from the Introduction



Case Representation Formalisms (III)

Object-Oriented Case Representation

- Refinement and more structured extension of attribute-value based representation
- Compositing of related attributes to object descriptions; each object is described by a fixed set of attributes → Case = Set of Objects

Graph- and Tree-Based Representation

- e.g. suited for atomic/molecule structures or electrical circuit designs

First-Order-Based Case Representation

- problems and solutions are represented as sets of *Grundatome* (variable-free)

Hierarchical Case Representation

- each case is represented on several levels of abstraction

Generalised Cases

- each case describes sets of cases at once, which are highly similar to one another → smaller case bases, simplified case/solution adaptation

3. SIMILARITY

When is a new problem (query) similar to a case's problem part?
What forms of similarity measures are suitable?

Meaning of Similarity

- Similarity is the **central notion** in Case-Based Reasoning.
- Similarity is always considered between problems (not solutions of cases).
- Selection of cases during the "Retrieve" phase is based on the similarity of cases to a given query.

→ **Observation I:** There is no universal similarity; similarity always relates to a certain purpose.

- e.g. two cars can be similar if they have the same max speed or cost approximately the same → different aspects of similarity

→ **Observation II:** Similarity is not necessarily transitive.

- e.g. 10€ are similar to 12€, 12€ are similar to 14€ ... 100€ are similar to 102€. But: 10€ are not similar to 102€ → property of "small numeric difference" is intransitive

→ **Observation III:** Similarity does not have to be symmetric.

Similarity and Utility

- Purpose of Similarity: Selection of solutions that can be easily transferred / adapted to the problem at hand.
- Similarity = Utility for Solving a (new) Problem**
- Note:
 - Utility is an a-posteriori criterion: In general, the utility (of a case) can be estimated **after** having solved the problem.
 - Similarity concerning problem situations is an a-priori criterion: Similarity must be estimated **before** solving the problem.
- Goal:** Similarity must approximate utility as accurately as possible.

Similarity Measures

- Idee:** Numerical modelling of similarity, capturing the degree of similarity

- Definition:** A **Similarity Measure** on a set M is a real-valued function $\text{sim}: M^2 \rightarrow [0, 1]$.

We say that sim is

- reflexive iff. $\forall x \in M: \text{sim}(x, x) = 1$
- symmetric iff. $\forall x, y \in M: \text{sim}(x, y) = \text{sim}(y, x)$

- Beyond ordinal information, similarity measures allow for a quantitative statement on the degree of similarity.

- Definition:** Each similarity measure induces a **similarity relation** R_{sim} as

$R_{\text{sim}}(x, y, u, v)$ iff. $\text{sim}(x, y) \geq \text{sim}(u, v)$

$y \succeq_x u$ iff. $\text{sim}(z, y) \geq \text{sim}(z, x)$



Distance Measures

- Definition:** A **Distance Measure** on a set M is a real-valued function $d: M^2 \rightarrow \mathbb{R}_0^+$. We say that sim is
 - reflexive iff. $\forall x \in M: d(x,x) = 0$
 - symmetric iff. $\forall x,y \in M: d(x,y) = d(y,x)$
- Definition:** A distance measure d on a set M is a **Metric** and (M,d) a **Metric Space** if
 - $\forall x,y \in M: d(x,y) = 0 \rightarrow x=y$
 - $\forall x,y,z \in M: d(x,y) + d(y,z) \geq d(x,z)$
- Definition:** Each distance induces a **similarity relation** R_d as
 - $\forall x,y,u,v \in M: R_d(x,y,u,v)$ iff. $d(x,y) \leq d(u,v)$
 - $\forall x,y,z \in M: y \geq z$ iff. $d(z,x) \leq d(z,y)$

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Relation Between Distance and Similarity Measures

- Definition:** A similarity measure sim and a distance measure d are called **Compatible** if and only if

$$\forall x,y,u,v \in M: R_{\text{sim}}(x,y,u,v) \leftrightarrow R_d(x,y,u,v)$$
- Lemma (Measure Transformation):** If there is a bijective, order-reversing mapping $f: \mathbb{R}_0^+ \rightarrow [0,1]$ with

$$f(0) = 1$$

$$f(d(x,y)) = \text{sim}(y,x)$$
 then sim and d are compatible.
- Note:** A transformation function f can be employed to construct a compatible pendant for a given sim or d , respectively.
- Examples:**
 - $f(x) = 1 - x/(x+1)$
 - $f(x) = 1 - x/x_{\max}$

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Exemplary Similarity Measures (I)

(for attribute-value based case representations)

Hamming Distance $H(x,y) = n - \sum_{i=1}^n x_i y_i - \sum_{i=1}^n (1-x_i)(1-y_i)$

- for binary features
- $x=(x_1, \dots, x_n), x_i \in \{0,1\}$
- $H(x,y) \in \{0, \dots, n\}$
- $H(x,y)$ is the number of attributes with differing values
- H is a distance measure: $H(x,x)=0, H(x,y)=H(y,x)$
- $H((x_1, \dots, x_n), (y_1, \dots, y_n)) = H((1-x_1, \dots, 1-x_n), (1-y_1, \dots, 1-y_n))$

SMC (Simple Matching Coefficient) $\text{SMC}(x,y) = 1 - \frac{n-(a+d)}{n} = \frac{a+d}{n} = 1 - \frac{b+c}{n}$

- $a = \sum x_i y_i, b = \sum x_i (1-y_i), c = \sum (1-x_i) y_i, d = \sum (1-x_i)(1-y_i)$
- $\rightarrow n = a+b+c+d \rightarrow H(x,y) = b+c = n - (a+d)$
- transformation into a compatible similarity measure by $f(x) = 1 - x/x_{\max}$ yields the simple matching coefficient

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Exemplary Similarity Measures (II)

(for attribute-value based case representations)

Measures for Real-Valued Attributes

- $x_i, y_i \in \mathbb{R}$ for all i
- generalisations of the Hamming distance
 - city-block metric d_1
 - Euclidean distance d_2
 - weighted Euclidean distance d_{2w}
 - p-norm d_p
- cf. lectures on *Vector Space & Linea Algebra*

Note: Similarity measures for other case representations (e.g. object-oriented, graph-based, etc.) are not considered in this lecture; see literature references.

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Local-Global Principle

- case description by n attributes A_1, \dots, A_n
- each attribute has a certain type T_i (e.g. numeric)
- Local Similarity**
 - a separate similarity function is used for each attribute: $\text{sim}_{A_i}: T_i \times T_i \rightarrow [0,1]$
 - local measures are depending on the respective type T_i of the attribute A_i
- Global Similarity**
 - $\text{sim}(x,y) = \text{sim}((x_1, \dots, x_n), (y_1, \dots, y_n)) = F(\text{sim}_{A_1}(x_1, y_1), \dots, \text{sim}_{A_n}(x_n, y_n))$
 - $F: [0,1]^n \rightarrow [0,1] \rightarrow$ **Amalgamation Function**
 - requirements on F :
 - F is monotonous in each of its arguments
 - $F(0, \dots, 0) = 0$ and $F(1, \dots, 1) = 1$

Very frequently used in practice!

Examples:

- Weighted Average $F(s_1, \dots, s_n) = \sum_{i=1}^n w_i s_i$
- Maximum $F(s_1, \dots, s_n) = \max\{s_1, \dots, s_n\}$
- k-Minimum $F(s_1, \dots, s_n) = s_k$ with $s_1 \leq s_2 \leq \dots \leq s_n$
- etc.

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Local Similarity Measures (I)

(for unordered symbolic and integer/real-valued attribute types)

Similarity Tables

- for attributes with symbolic type $T_A = \{v_1, \dots, v_k\}$
- sim table/matrix $\text{sim}_A(x,y) = s[x,y]$
- example: attribute "RAM-Type" with $T_A = \{SD, DDR, RD\}$

\ c	SD	DDR	RD
SD	1.0	0.9	0.75
DDR	0.5	1.0	0.75
RD	0.25	0.5	1.0

RAM-Type

- reflexive similarity measure: diagonal elements $s[k,k]=1$
- symmetric similarity measure: $s = s^T$

Difference-Based Similarity Functions

- for attributes with numeric type (e.g. integer or real-valued)
- similarity is based on the numerical difference between case and query value
 - linearly scaled domains: $\text{sim}_A(x,y) = f(x-y)$
 - exponentially scaled domains: $\text{sim}_A(x,y) = f(\log(x) - \log(y))$
- typical requirements on f
 - $f: \mathbb{R} \rightarrow [0,1]$ or $\mathbb{Z} \rightarrow [0,1]$
 - $f(0)=1$ (reflexivity)
 - $f(x)$ is monotonously falling/increasing
- examples: next slide

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Local Similarity Measures (II)

(for unordered symbolic and integer/real-valued attribute types)

- symmetric
- simple distance
- asymmetric
- $x = \text{query } q, y = \text{case } c$
- query is minimal requirement
- e.g. minimal horse power of a car required by a customer
- asymmetric
- $x = \text{query } q, y = \text{case } c$
- query depicts maximal value
- e.g. maximal price of a product included (for $q-c > 0$)

Local Similarity for Other Types → not considered here

- ordered symbolic data types
 - e.g. $T_A = \{\text{small, average, tall}\}$
- taxonomic data types
 - elements of T_A can be arranged within a taxonomical (tree) structure
 - e.g. attribute to describe the types of CPUs

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4. SIMILARITY-BASED RETRIEVAL

How to retrieve a query's nearest neighbour(s)?

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Sequential Retrieval

- **Retrieval Task**
 - Input
 - case base $CB = \{c_1, \dots, c_n\}$
 - similarity measure sim
 - query (new problem) q
 - Output
 1. most similar case c_i
 - or
 2. m most similar cases $\{c_1, \dots, c_m\}$
 - or
 3. all cases $\{c_1, \dots, c_n\}$ which have at least a similarity of sim_{\min} to q
 - Main Problem: Efficiency
 - Question: How can the case base be organised in such a way to support an efficient retrieval?
- **Sequential Retrieval**
 - iterates over all $c \in CB$ and calculates $\text{sim}(c, q)$
 - returns the most similar / m most similar cases to q
 - complexity: $O(n)$
 - Advantages
 - easy to implement
 - no index structures to maintain
 - usability of arbitrary similarity measures
 - Drawbacks
 - problematic for large case bases
 - effort independent of query
 - effort independent of m

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Two-Stage Retrieval

- Idea: **MAC/FAC** (many are called, few are chosen)
 1. preselection of possible solution candidates $M_q \subseteq CB$, where $M_q = \{c \in CB \mid \text{fac}(q, c)\}$
 2. use sequential retrieval on M_q

→ **Problem:** Finding of an adequate predicate fac
- Examples for predicate fac
 - partial equality: $\text{fac}(q, c)$ iff. q and c are identical w.r.t. at least one attribute
 - local similarity: $\text{fac}(q, c)$ iff. q and c are sufficiently similar w.r.t. to each attribute
 - partial local similarity: $\text{fac}(q, c)$ iff. q and c are sufficiently similar w.r.t. to at least one attribute
- Advantage: good performance $|M_q|$ if is small
- Drawbacks
 - retrieval errors may occur → α -error: A case c that is sufficiently similar to q w.r.t. sim is not found (because not considered during preselection).
 - completeness of retrieval is not guaranteed
 - determination of an adequate predicate for preselection is usually difficult

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Case Retrieval with kd-Trees

- **Index-Oriented Retrieval Procedures**
 - preprocessing: generating an index structure
 - retrieval: exploit the index structure to efficiently access the cases
- Possible Index Structure: kd-Tree
 - A **kd-Tree** is a k -dimensional binary search tree to support an efficient search over data sets.
 - Idee: partitioning of the data (here: the case base) into small intervals.
 - ordering within a binary tree (similar to a decision tree)
 - during retrieval
 - stepping through the tree from root to the leaves
 - backtracking is possible (unlike in decision trees)

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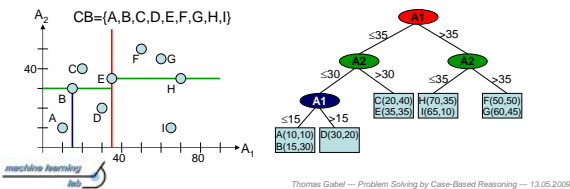
Definition of kd-Trees

- Input
 - k ordered domains T_1, \dots, T_k for attributes A_1, \dots, A_k
 - case base $CB \subseteq T_1 \times \dots \times T_k$
 - parameter b (bucket size)
- **Definition:** A **kd-Tree** $T(CB)$ for case base CB is a binary tree, that is defined as
 - if $|CB| \leq b$: $T(CB)$ is a leaf of the tree (called bucket), denoted CB
 - if $|CB| > b$: $T(CB)$ is a tree whose
 - root is denoted with an attribute A_i and a value $v_i \in T_i$ and
 - two sub-trees $T_s(CB_s)$ and $T_r(CB_r)$ are kd-trees, too, with
 - $CB_s := \{(x_1, \dots, x_k) \in CB \mid x_i \leq v_i\}$ and
 - $CB_r := \{(x_1, \dots, x_k) \in CB \mid x_i > v_i\}$

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Properties of kd-Trees

- kd-tree partitions the case base
 - root represents the entire case base
 - a leaf (bucket) represents a subset of the case base that does not have to be further partitioned
 - at each inner node the case base is partitioned, being divided on the basis of some specific value of an attribute
- Example:



Generating kd-Trees

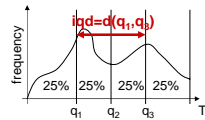
- Algorithm

```

PROCEDURE CreateTree(CB): kd-Tree
if |CB|<b
then
    return leaf node marked with case base CB
else
    Ai := choose_attribute(CB)
    vi := chose_split_value(CB,Ai)
    return
        tree whose root is marked with Ai and vi
        and which has sub-trees
        CreateTree( {(x1,...,xk)∈CB | xi≤vi} )
        CreateTree( {(x1,...,xk)∈CB | xi>vi} )
    
```

Attribute Selection and Splitting Values

- various methods usable for attribute selection
 - entropy-based
 - inter-quartile distance
 - choose the attribute with the biggest inter-quartile distance **iqd**
- determination of splitting values
 - median splitting: choose median as splitting value
 - maximum splitting: search for the "largest gap"



Retrieval Algorithm Using kd-Trees

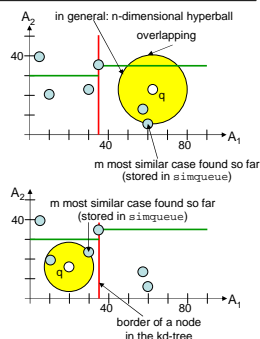
- Algorithm
 - simqueue is a global data structure holding the m most similar cases as well as their corresponding similarities with respect to q

```

PROCEDURE Retrieve(K: kd-Tree, q: Query)
if K is leaf node
then
    forall c∈CB do
        if sim(q,c)> simqueue[m].sim
            then insert c into simqueue
else
    Ai := the attribute K is marked with
    vi := the splitting value K is marked with
    qi := the attribute Ai's value of q
    if qi≤vi
        then
            Retrieve(K1)
            if BOB-Test is fulfilled then Retrieve(K2)
        else
            Retrieve(K2)
            if BOB-Test is fulfilled then Retrieve(K1)
    if BWB-Test is fulfilled
        then terminate retrieval returning simqueue
    else return
    
```

BOB- and BWB-Tests

- BOB-Test:** Can there be – in the neighbouring sub-tree – any more similar cases (to query q) than the m most similar cases already found?
 - in general: n-dimensional hyperball overlapping
 - m most similar case found so far (stored in simqueue)
- BWB-Test:** Is it guaranteed that there is no case in a neighbouring sub-tree which is more similar to the query q than the m-most similar case found so far?
 - border of a node in the kd-tree



Discussion of kd-Tree Retrieval

- Restriction**
 - Retrieval using kd-trees guarantees finding the m nearest neighbours, if the similarity measure used fulfills the following condition:
 - Compatibility with ordering and monotony:

$$\forall x_1, \dots, x_n \text{ and } x'_1, x'_2 \text{ if } x_1 < x'_1 < x'_2 \text{ then } \text{sim}(x_1, \dots, x_n), (x_1, \dots, x'_1, \dots, x_n) \geq \text{sim}(x_1, \dots, x_n), (x_1, \dots, x'_2, \dots, x_n)$$
- Advantages**
 - efficient retrieval
 - effort depends on the number m of most similar cases to find
 - incremental extension of the kd-tree is possible
- Drawbacks**
 - higher costs for building up the index structure (kd-tree)
 - restrictions implied by kd-trees
 - usability for ordered domains only
 - unknown attribute values are difficult to handle
 - only for monotonous similarity measures that are compatible with the ordering of the respective attribute's domain

Other Retrieval Methods

- Several further advanced retrieval approaches
 - high efficiency
 - general usability depends on problem setting (e.g. case modelling)
- Examples
 - Case Retrieval Nets [Burkhardt&Lenz]
 - Retrieval with ``Fish and Shrink`` [Schaaf, 1996]
 - Case Retrieval on Top of Relational Databases Utilising SQL [Schumacher, 2000]

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Part 1 (overed today)

Part 2: Outlook

1. Introduction
(What is CBR?)

2. Knowledge and Case Representation
(What knowledge is in a CBR system? How can cases be represented?)

3. Similarity
(When is a new problem similar to an old one? What types of similarity measure may be used?)

4. Similarity-Based Retrieval
(How to retrieve a query's nearest neighbors?)

5. SOLUTION ADAPTATION
How to adapt existing solutions to be applicable for the problem at hand?

6. LEARNING IN CASE-BASED REASONING
Where are the explicit links between CBR and Machine Learning?

7. APPLICATIONS AND TOOLS
Is CBR actually employed in practice? Are there tools available I may use for trying out some of the things introduced in this talk?

8. REFERENCES
Where can I find more about CBR?

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Thanks!

Questions?

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