Machine Learning

Reinforcement Learning

Prof. Dr. Martin Riedmiller
AG Maschinelles Lernen und Natürlichsprachliche Systeme
Institut für Informatik
Technische Fakultät
Albert-Ludwigs-Universität Freiburg

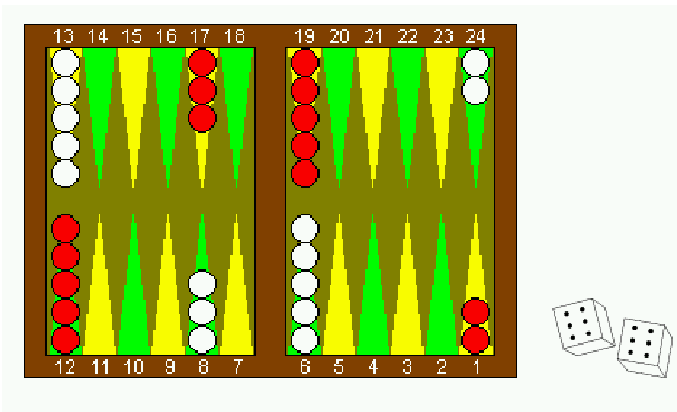
Martin.Riedmiller@informatik.uni-freiburg.de

Motivation

Can a software agent learn to play Backgammon by itself?

Motivation

Can a software agent learn to play Backgammon by itself?

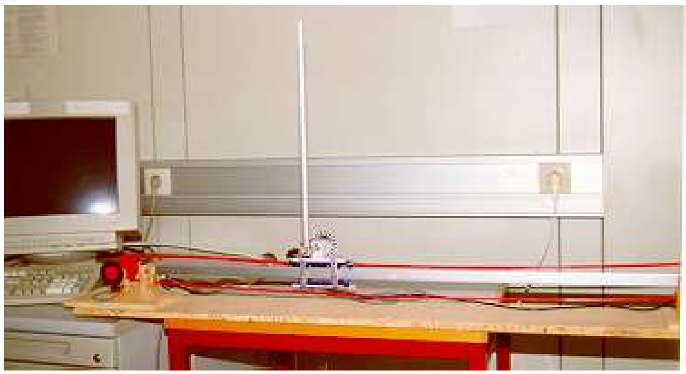


Learning from success or failure

Neuro-Backgammon:
playing at worldchampion level (Tesauro, 1992)

Can a software agent learn to balance a pole by itself?

Can a software agent learn to balance a pole by itself?

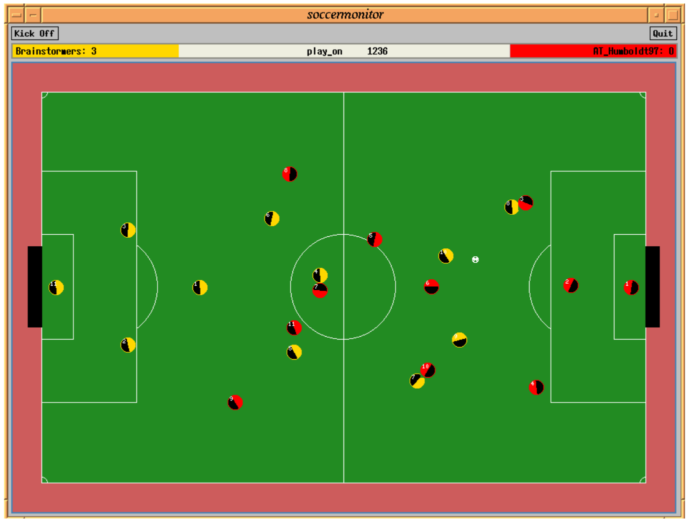


Learning from success or failure

Neural RL controllers:
noisy, unknown, nonlinear (Riedmiller et.al.
)

Can a software agent learn to cooperate with others by itself?

Can a software agent learn to cooperate with others by itself?



Learning from success or failure

Cooperative RL agents:
complex, multi-agent, cooperative
(Riedmiller et.al.)

Reinforcement Learning

has biological roots: reward and punishment

no teacher, but:

Reinforcement Learning

has biological roots: reward and punishment

no teacher, but:

actions

Reinforcement Learning

has biological roots: reward and punishment

no teacher, but:

actions $+$ goal

Reinforcement Learning

has biological roots: reward and punishment

no teacher, but:

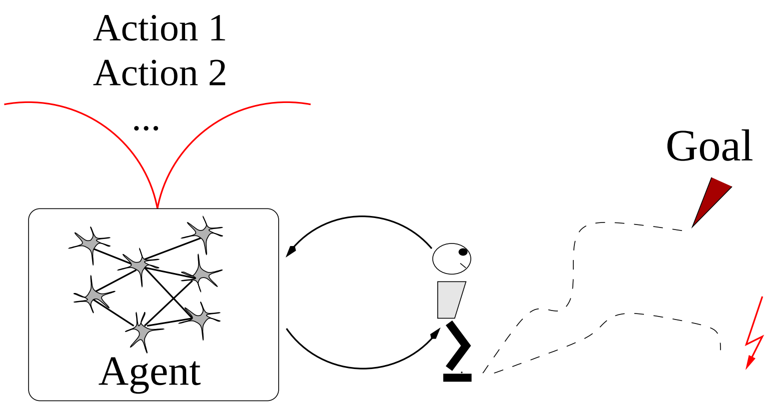
actions $+$ goal $\stackrel{learn}{\rightarrow}$ algorithm/ policy

Reinforcement Learning

has biological roots: reward and punishment

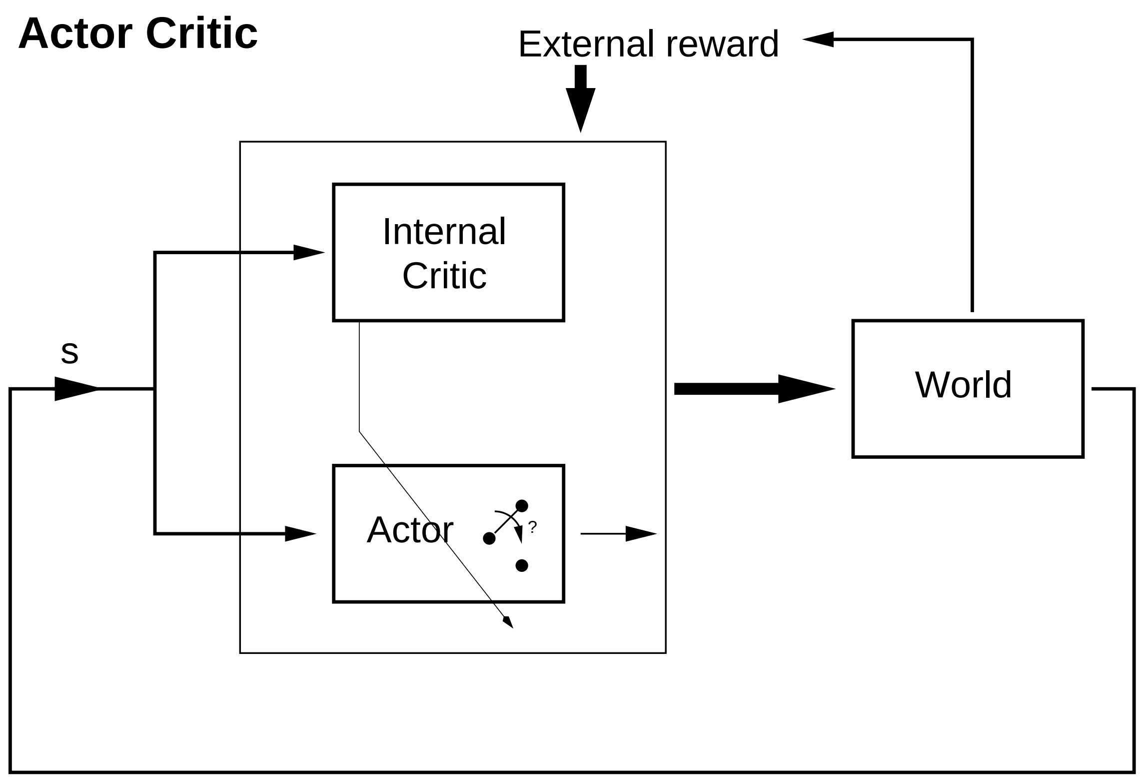
no teacher, but:

actions + goal $\stackrel{learn}{\longrightarrow}$ algorithm/ policy



'Happy Programming'

Actor-Critic Scheme (Barto, Sutton, 1983)



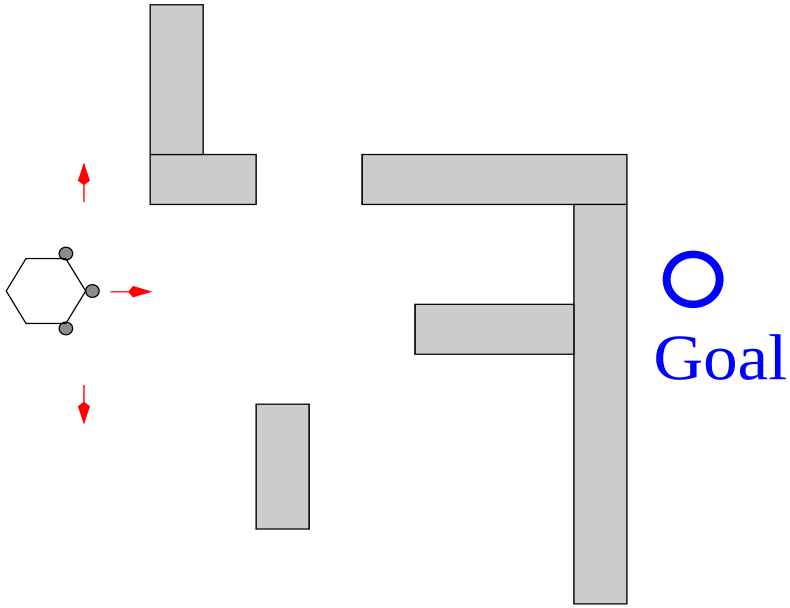
ACTOR-CRITIC SCHEME:

- Critic maps external, delayed reward in internal training signal
- Actor represents policy

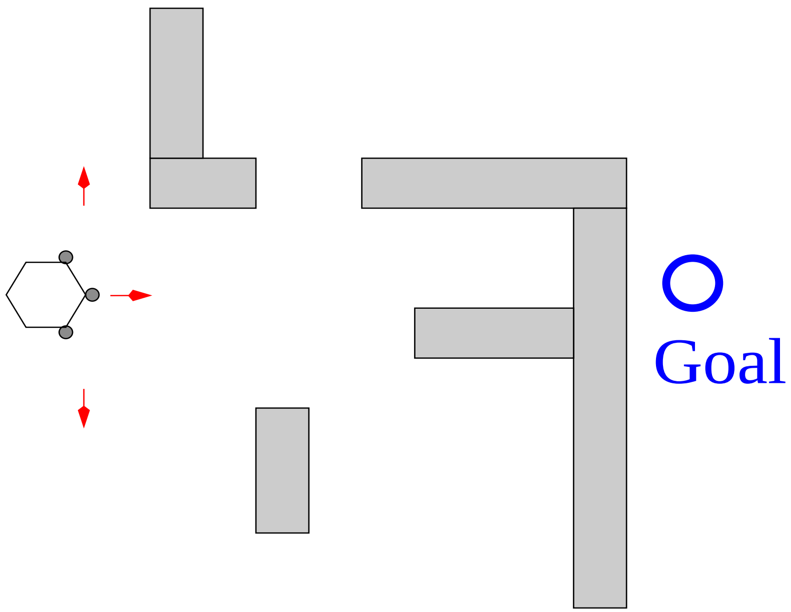
Overview

I Reinforcement Learning - Basics

A First Example



A First Example

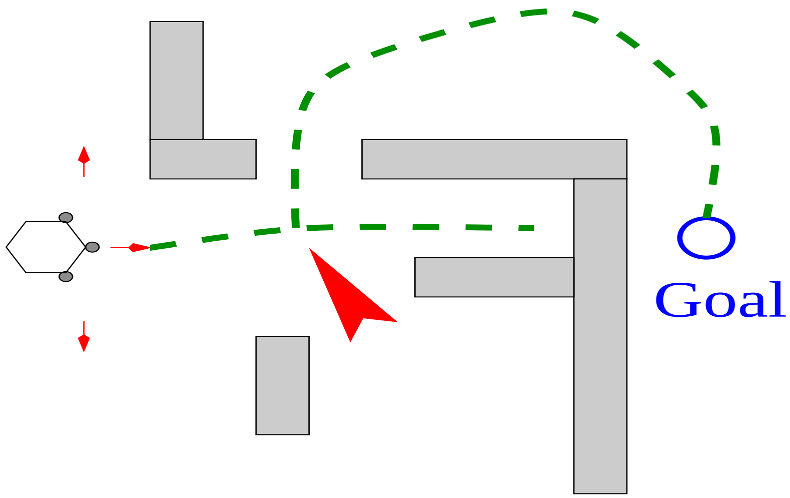


REPEAT

Choose: Action $a \in \{\rightarrow, \leftarrow, \uparrow\}$

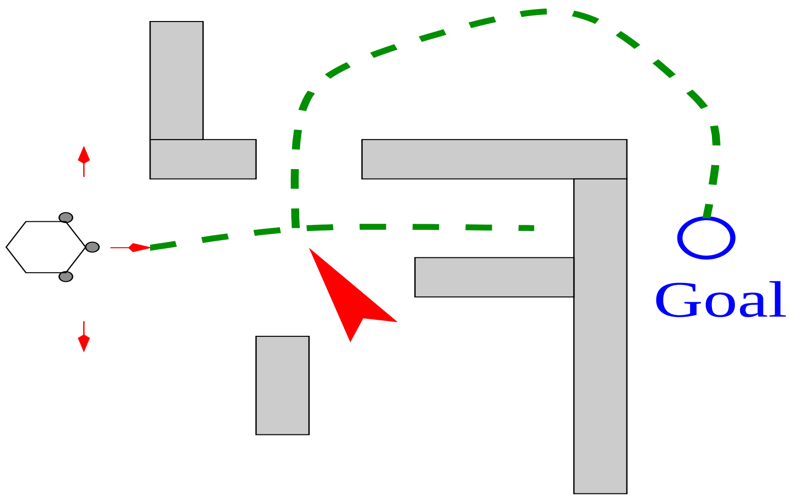
UNTIL Goal is reached

The 'Temporal Credit Assignment' Problem



Which action(s) in the sequence has to be changed?

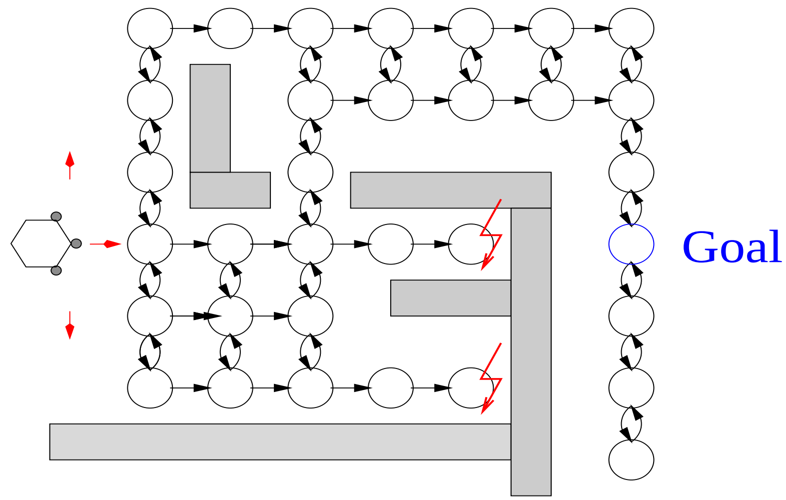
The 'Temporal Credit Assignment' Problem



Which action(s) in the sequence has to be changed?

⇒ Temporal Credit Assignment Problem

Sequential Decision Making



Examples:
Chess, Checkers (Samuel, 1959), Backgammon (Tesauro, 92)
Cart-Pole-Balancing (AHC/ ACE (Barto, Sutton, Anderson, 1983)), Robotics and control, . . .

Three Steps

Three Steps

⇒ Describe environment as a Markov Decision Process (MDP)

Three Steps

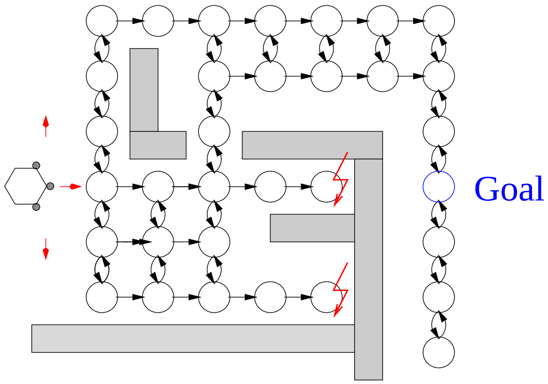
$\Rightarrow$ Describe environment as a Markov Decision Process (MDP)

$\Rightarrow$ Formulate learning task as a dynamic optimization problem

Three Steps

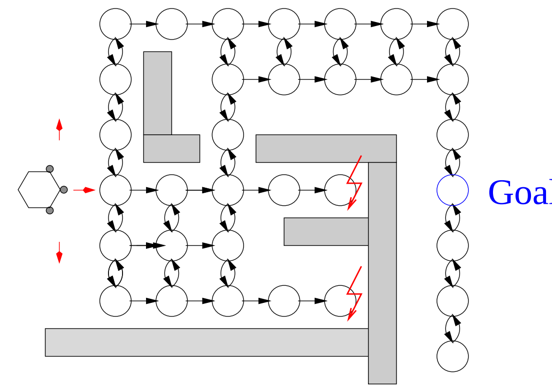
- ⇒ Describe environment as a Markov Decision Process (MDP)
- ⇒ Formulate learning task as a dynamic optimization problem
- ⇒ Solve dynamic optimization problem by dynamic programming methods

1. Description of the environment



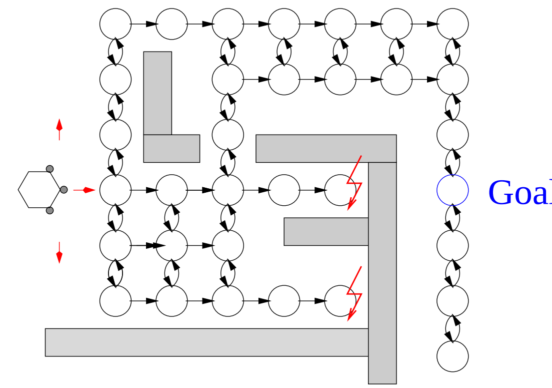
1. Description of the environment

S: (finite) set of states

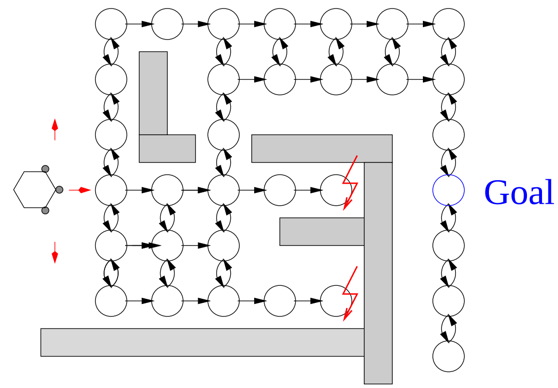


1. Description of the environment

S: (finite) set of states
A: (finite) set of actions



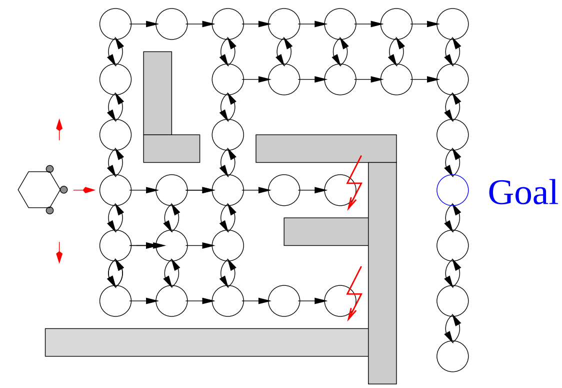
1. Description of the environment



S: (finite) set of states
A: (finite) set of actions

Behaviour of the environment 'model'
$p : S \times S \times A \to [0, 1]$
$p(s', s, a)$ Probability distribution of transition

1. Description of the environment

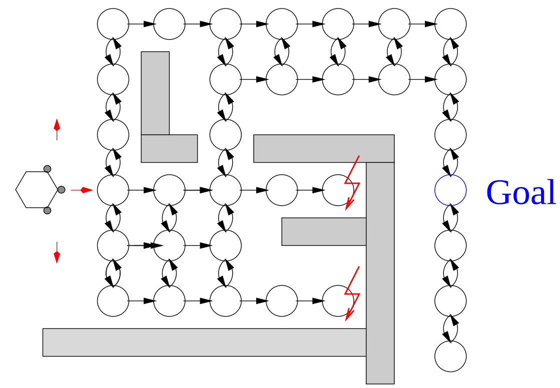


S: (finite) set of states
A: (finite) set of actions

Behaviour of the environment 'model'
$p : S \times S \times A \rightarrow [0, 1]$
$p(s', s, a)$ Probability distribution of transition

For simplicity, we will first assume a deterministic environment. There, the model can be described by a transition function
$f : S \times A \rightarrow S$, $s' = f(s, a)$

1. Description of the environment



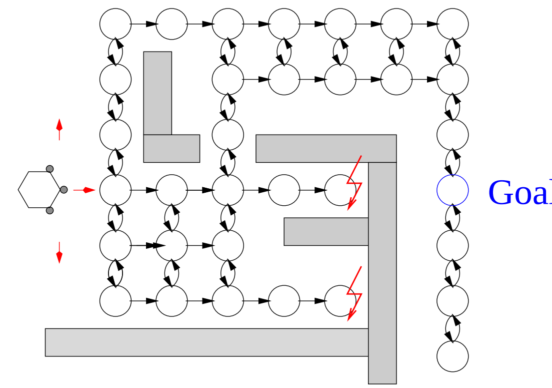
S: (finite) set of states
A: (finite) set of actions

Behaviour of the environment 'model'
$p : S \times S \times A \rightarrow [0, 1]$
$p(s', s, a)$ Probability distribution of transition

For simplicity, we will first assume a deterministic environment. There, the model can be described by a transition function
$f : S \times A \rightarrow S$, $s' = f(s, a)$

'Markov' property: Transition only depends on current state and action

1. Description of the environment



S: (finite) set of states
A: (finite) set of actions

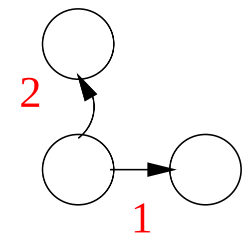
Behaviour of the environment 'model'
$p : S \times S \times A \rightarrow [0, 1]$
$p(s', s, a)$ Probability distribution of transition

For simplicity, we will first assume a deterministic environment. There, the model can be described by a transition function
$f : S \times A \rightarrow S$, $s' = f(s, a)$

'Markov' property: Transition only depends on current state and action
$Pr(s_{t+1}|s_t, a_t) = Pr(s_{t+1}|s_t, a_t, s_{t-1}, a_{t-1}, s_{t-2}, a_{t-2}, \ldots)$

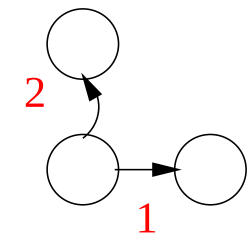
2. Formulation of the learning task

every transition emits transition costs, 'immediate costs', $c : S \times A \longrightarrow \Re$
(sometimes also called 'immediate reward', r)

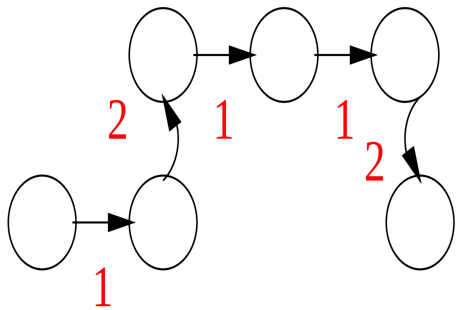


2. Formulation of the learning task

every transition emits transition costs, 'immediate costs', $c : S \times A \longrightarrow \Re$
(sometimes also called 'immediate reward', r)

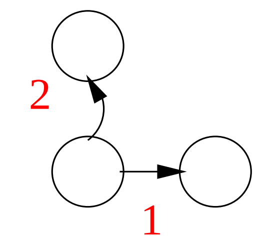


Now, an agent policy $\pi : S \rightarrow A$ can be evaluated (and judged):
Consider pathcosts:
$J^\pi(s) = \sum_t c(s_t, \pi(s_t)), s_0 = s$

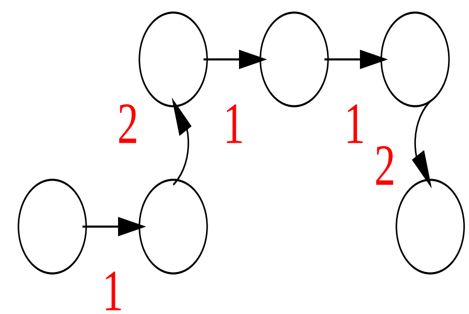


2. Formulation of the learning task

every transition emits transition costs, 'immediate costs', $c : S \times A \rightarrow \Re$
(sometimes also called 'immediate reward', r)



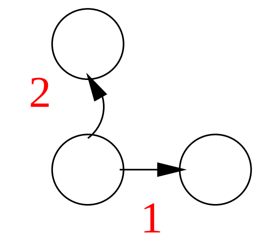
Now, an agent policy $\pi : S \rightarrow A$ can be evaluated (and judged):
Consider pathcosts:
$J^{\pi}(s) = \sum_t c(s_t, \pi(s_t)), s_0 = s$



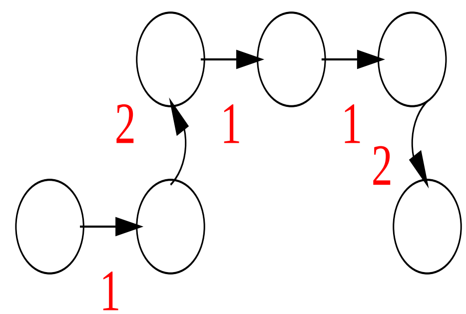
Wanted: optimal policy $\pi^* : \mathcal{S} \rightarrow \mathcal{A}$
where $J^{\pi^*}(s) = \min_{\pi}\{\sum_t c(s_t, \pi(s_t)) | s_0 = s\}$

2. Formulation of the learning task

every transition emits transition costs, 'immediate costs', $c : S \times A \rightarrow \Re$ (sometimes also called 'immediate reward', r)



Now, an agent policy $\pi : S \rightarrow A$ can be evaluated (and judged):
Consider pathcosts:
$J^\pi(s) = \sum_t c(s_t, \pi(s_t)), s_0 = s$

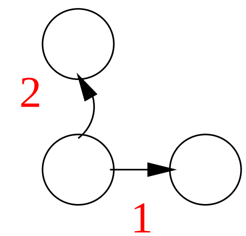


Wanted: optimal policy $\pi^* : \mathcal{S} \rightarrow \mathcal{A}$
where $J^{\pi^*}(s) = \min_\pi \{\sum_t c(s_t, \pi(s_t)) | s_0 = s\}$

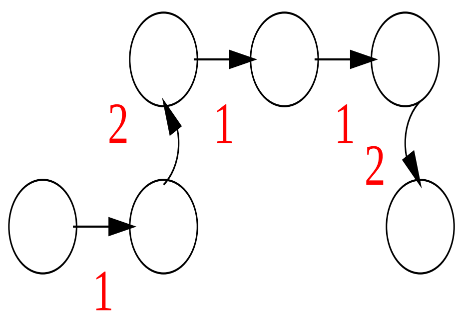
$\Rightarrow$ Additive (path-)costs allow to consider *all* events

2. Formulation of the learning task

every transition emits transition costs, 'immediate costs', $c : S \times A \to \Re$
(sometimes also called 'immediate reward', r)



Now, an agent policy $\pi : S \to A$ can be evaluated (and judged):
Consider pathcosts:
$J^\pi(s) = \sum_t c(s_t, \pi(s_t)), s_0 = s$



Wanted: optimal policy $\pi^* : \mathcal{S} \to \mathcal{A}$
where $J^{\pi^*}(s) = \min_\pi \{\sum_t c(s_t, \pi(s_t)) | s_0 = s\}$

$\Rightarrow$ Additive (path-)costs allow to consider *all* events

$\Rightarrow$ Does this solve the temporal credit assignment problem? YES!

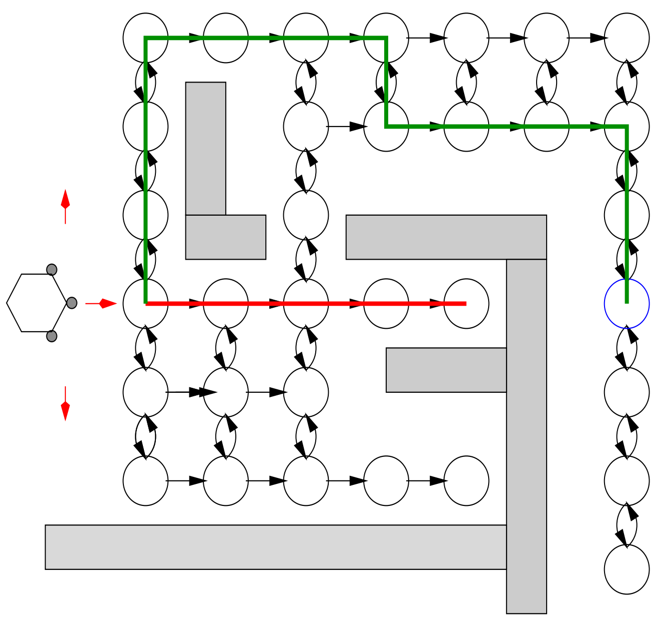
Choice of immediate cost function $c(\cdot)$ specifies policy to be learned

$J^{\pi}(s_{start}) = 12$

Example:

$J^{\pi}(s_{start}) = 1004$

$$c(s) = \begin{cases} 0 & , \quad \text{if } s \text{ success } (s \in \color{red}{Goal}) \\ 1000 & , \quad \text{if } s \text{ failure } (s \in \color{red}{Failure}) \\ 1 & , \quad else \end{cases}$$



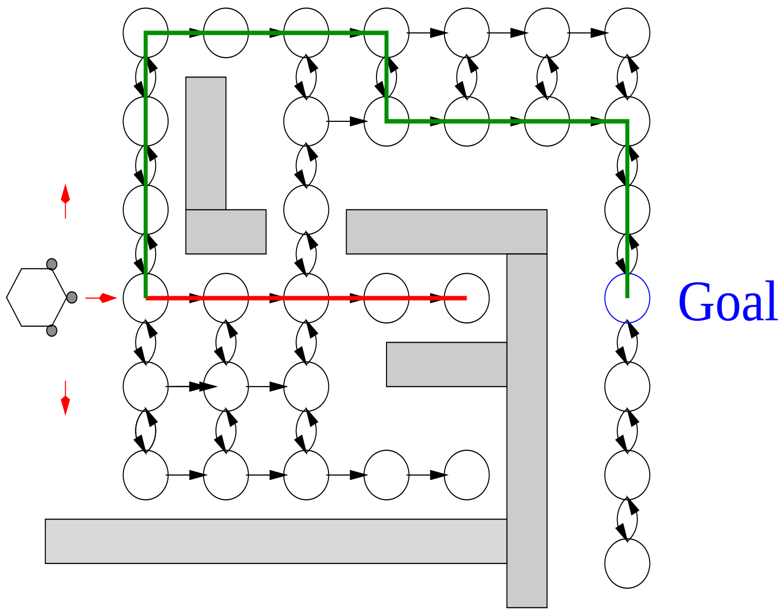
$J^{\pi}(s_{start}) = 12$

Choice of immediate cost function $c(\cdot)$ specifies policy to be learned

$J^{\pi}(s_{start}) = 1004$

Example:

$$c(s) = \begin{cases} 0 & , \quad \text{if } s \text{ success } (s \in Goal) \\ 1000 & , \quad \text{if } s \text{ failure } (s \in Failure) \\ 1 & , \quad else \end{cases}$$



$\Rightarrow$ specification of requested policy by $c(\cdot)$ is simple!

3. Solving the optimization problem

For the optimal path costs it is known that

$$J^*(s) = \min_a \{c(s,a) + J^*(f(s,a))\}$$

(Principle of Optimality (Bellman, 1959))

$\Rightarrow$Can we compute J^* (we will see why, soon)?

Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$

Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$

for all states s :

Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$

for all states s : $J_{k+1}(s) :=$

Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$
for all states s : $J_{k+1}(s) := \min_{a \in \mathcal{A}}$

Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$
for all states s : $J_{k+1}(s) := \min_{a \in \mathcal{A}} \{c(s,a)$

Computing J^*: the value iteration (VI) algorithm

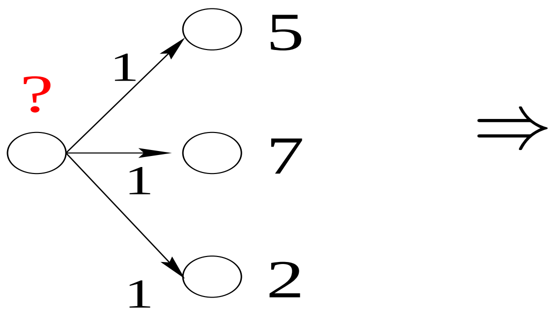
Start with arbitrary $J_0(s)$
for all states s : $J_{k+1}(s) := \min_{a \in \mathcal{A}}\{c(s,a) + J_k()\}$

Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$
for all states s : $J_{k+1}(s) := \min_{a \in \mathcal{A}} \{c(s,a) + J_k(f(s,a))\}$

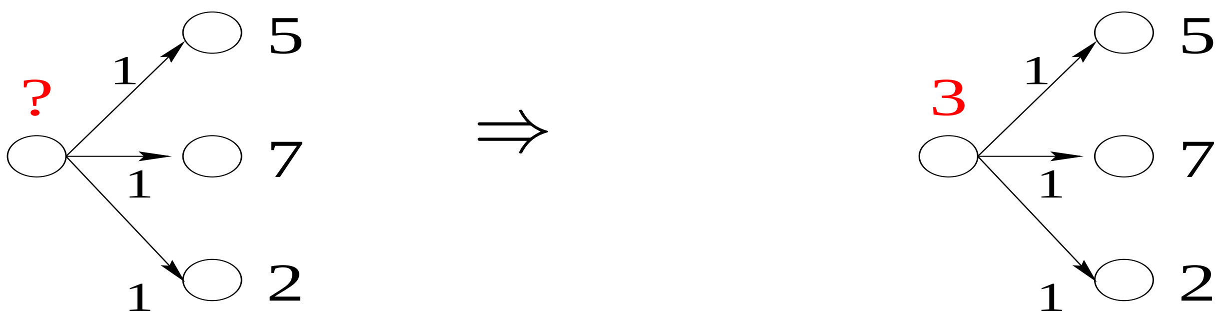
Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$
for all states s : $J_{k+1}(s) := \min_{a \in \mathcal{A}}\{c(s,a) + J_k(f(s,a))\}$



Computing J^*: the value iteration (VI) algorithm

Start with arbitrary $J_0(s)$
for all states s : $J_{k+1}(s) := \min_{a \in \mathcal{A}} \{c(s,a) + J_k(f(s,a))\}$



Convergence of value iteration

Value iteration converges under certain assumptions, i.e. we have $lim_{k \rightarrow \infty} J_k = J^*$

Convergence of value iteration

Value iteration converges under certain assumptions, i.e. we have $lim_{k \to \infty} J_k = J^*$

$\Rightarrow$ Discounted problems: $J^{\pi^*}(s) = \min_\pi \{\sum_t \gamma^t c(s_t, \pi(s_t)) | s_0 = s\}$ where $0 \leq \gamma < 1$ (contraction mapping)

Convergence of value iteration

Value iteration converges under certain assumptions, i.e. we have $lim_{k\to\infty} J_k = J^*$

$\Rightarrow$ Discounted problems: $J^{\pi^*}(s) = \min_\pi\{\sum_t \gamma^t c(s_t, \pi(s_t))|s_0 = s\}$ where $0 \leq \gamma < 1$ (contraction mapping)

$\Rightarrow$ Stochastic shortest path problems:

- there exists an absorbing terminal state with zero costs

Convergence of value iteration

Value iteration converges under certain assumptions, i.e. we have $lim_{k \to \infty} J_k = J^*$

$\Rightarrow$ Discounted problems: $J^{\pi^*}(s) = \min_\pi \{\sum_t \gamma^t c(s_t, \pi(s_t)) | s_0 = s\}$ where $0 \leq \gamma < 1$ (contraction mapping)

$\Rightarrow$ Stochastic shortest path problems:

- there exists an absorbing terminal state with zero costs
- there exists a 'proper' policy (a policy that has a non-zero chance to finally reach the terminal state)

Convergence of value iteration

Value iteration converges under certain assumptions, i.e. we have $lim_{k \to \infty} J_k = J^*$

$\Rightarrow$ Discounted problems: $J^{\pi^*}(s) = \min_\pi \{\sum_t \gamma^t c(s_t, \pi(s_t)) | s_0 = s\}$ where $0 \leq \gamma < 1$ (contraction mapping)

$\Rightarrow$ Stochastic shortest path problems:

- there exists an absorbing terminal state with zero costs
- there exists a 'proper' policy (a policy that has a non-zero chance to finally reach the terminal state)
- every non-proper policy has infinite path costs for at least one state

Ok, now we have J^*

Ok, now we have J^*

$\Rightarrow$ when J^* is known, then we also know an optimal policy:

Ok, now we have J^*

$\Rightarrow$ when J^* is known, then we also know an optimal policy:

$$\pi^*(s) \quad \in \quad \arg\min_{a \in \mathcal{A}} \{\}$$

Ok, now we have J^*

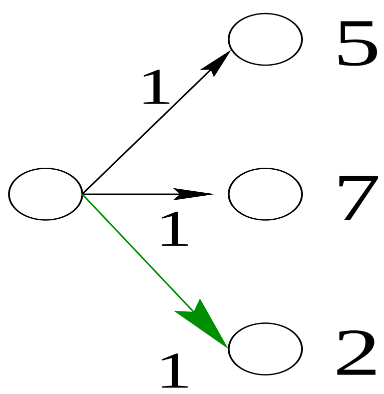
$\Rightarrow$ when J^* is known, then we also know an optimal policy:

$$\pi^*(s) \quad \in \quad \arg\min_{a\in\mathcal{A}}\{c(s,a) + J^*(f(s,a))\}$$

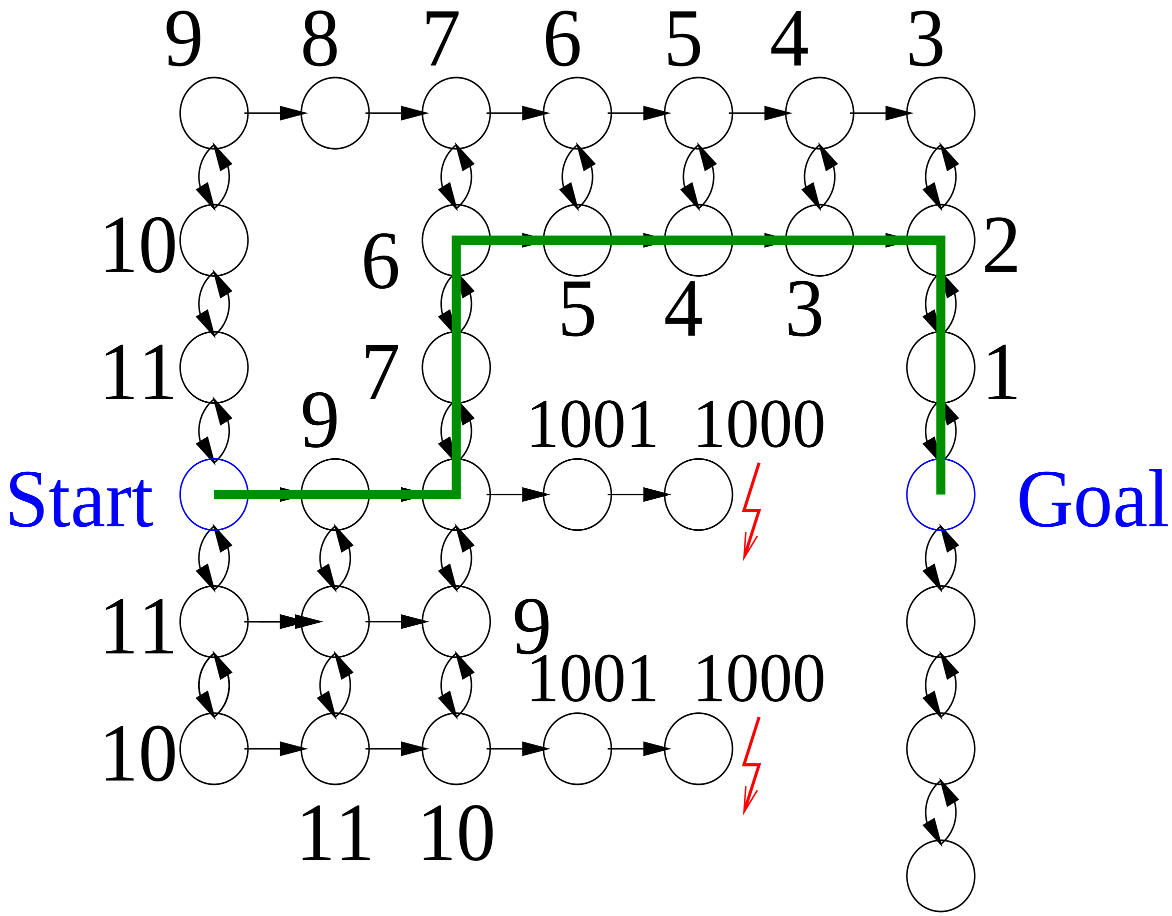
Ok, now we have J^*

$\Rightarrow$ when J^* is known, then we also know an optimal policy:

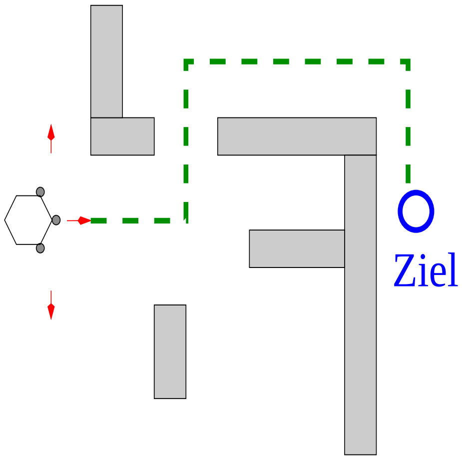
$$\pi^*(s) \quad \in \quad \arg\min_{a \in \mathcal{A}} \{c(s,a) + J^*(f(s,a))\}$$



Back to our maze

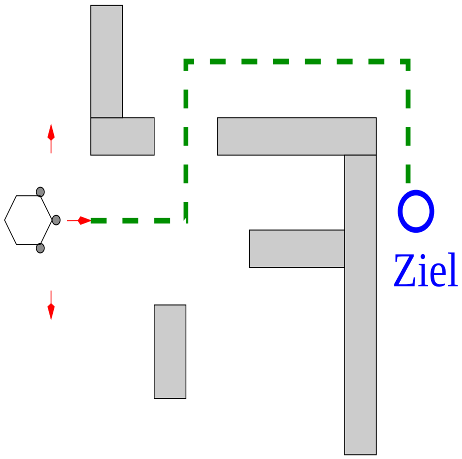


Overview of the approach so far

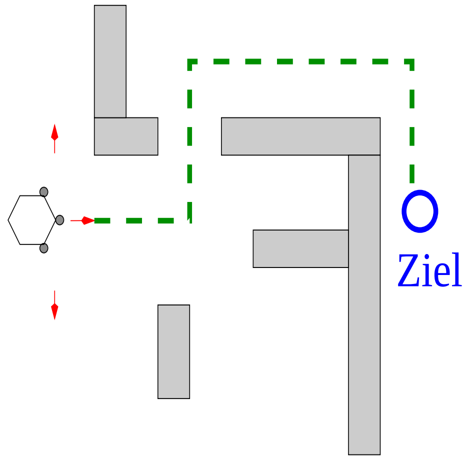


Overview of the approach so far

- Description of the learning task as an MDP
  S, A, T, f, c
  c specifies requested behaviour/ policy



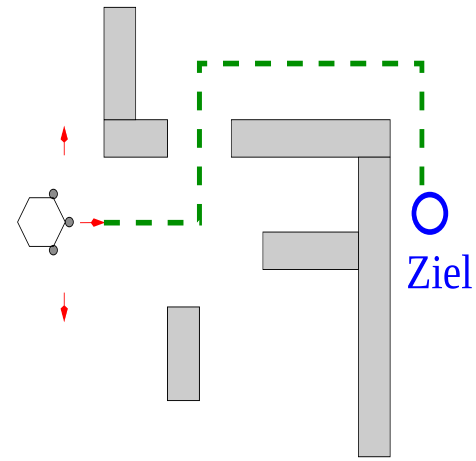
Overview of the approach so far



- Description of the learning task as an MDP
  S, A, T, f, c
  c specifies requested behaviour/ policy

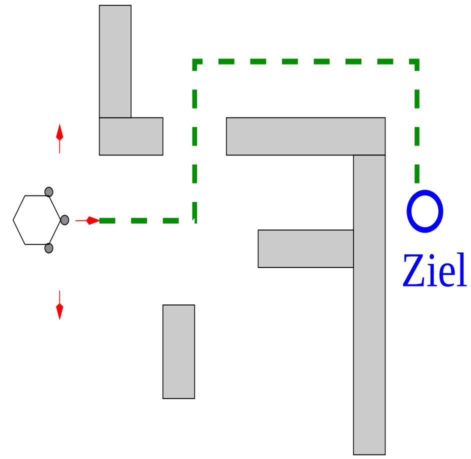
- iterative computation of optimal pathcosts J^*:
  $\forall s \in \mathcal{S} : J_{k+1}(s) = \min_{a \in \mathcal{A}} \{c(s,a) + J_k(f(s,a))\}$

Overview of the approach so far



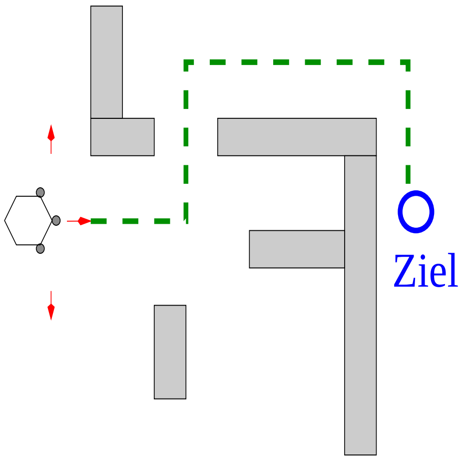
- Description of the learning task as an MDP
  S, A, T, f, c
  c specifies requested behaviour/ policy
- iterative computation of optimal pathcosts J^*:
  $\forall s \in \mathcal{S} : J_{k+1}(s) = \min_{a \in \mathcal{A}} \{c(s,a) + J_k(f(s,a))\}$
- Computation of an optimal policy from J^*
  $\pi^*(s) \in \arg\min_{a \in \mathcal{A}} \{c(s,a) + J^*(f(s,a))\}$

Overview of the approach so far

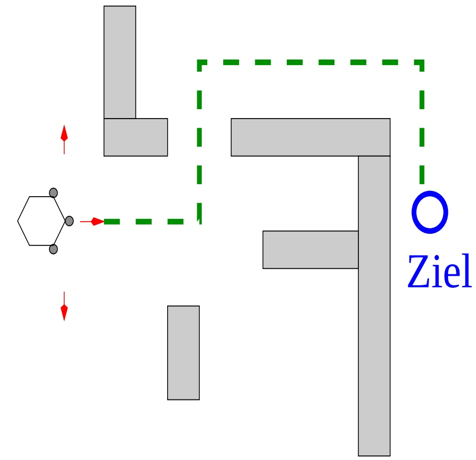


- Description of the learning task as an MDP
  S, A, T, f, c
  c specifies requested behaviour/ policy
- iterative computation of optimal pathcosts J^*:
  $\forall s \in \mathcal{S} : J_{k+1}(s) = \min_{a \in \mathcal{A}} \{c(s,a) + J_k(f(s,a))\}$
- Computation of an optimal policy from J^*
  $\pi^*(s) \in \arg\min_{a \in \mathcal{A}} \{c(s,a) + J^*(f(s,a))\}$
- value function ('costs-to-go') can be stored in a table

Overview of the approach: Stochastic Domains

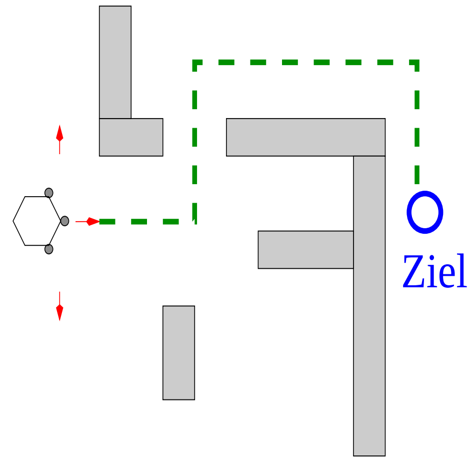


Overview of the approach: Stochastic Domains



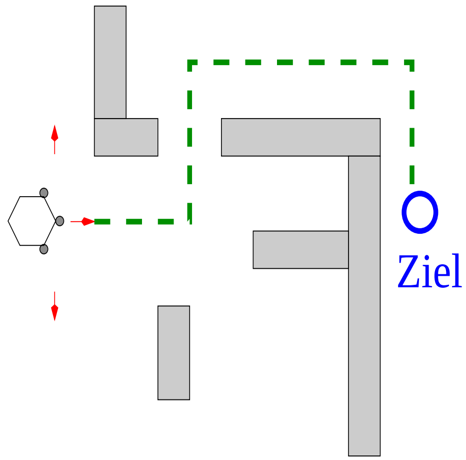
- value iteration in stochastic environments:
  $\forall s \in \mathcal{S} : J_{k+1}(s) = \min_{a \in \mathcal{A}}\{\,(c(s,a) + J_k(s'))\}$

Overview of the approach: Stochastic Domains



- value iteration in stochastic environments:
  $\forall s \in \mathcal{S} : J_{k+1}(s) = \min_{a \in \mathcal{A}} \{ \sum_{s' \in S} p(s, s', a) \, (c(s,a) + J_k(s')) \}$

Overview of the approach: Stochastic Domains



- value iteration in stochastic environments:
  $\forall s \in \mathcal{S} : J_{k+1}(s) = \min_{a \in \mathcal{A}} \{ \sum_{s' \in S} p(s, s', a) \, (c(s, a) + J_k(s')) \}$
- Computation of an optimal policy from J^*
  $\pi^*(s) \in \arg\min_{a \in \mathcal{A}} \{ \sum_{s' \in S} p(s, s', a) \, (c(s, a) + J_k(s')) \}$
- value function J ('costs-to-go') can be stored in a table

Reinforcement Learning

Problems of Value Iteration:

Reinforcement Learning

Problems of Value Iteration:

for all $s \in \mathcal{S}$:

Reinforcement Learning

Problems of Value Iteration:

for all $s \in \mathcal{S}$:$J_{k+1}(s) = \min_{a \in \mathcal{A}} \{c(s,a) + J_k()\}$

Reinforcement Learning

Problems of Value Iteration:

for all $s \in \mathcal{S}$:$J_{k+1}(s) = \min_{a \in \mathcal{A}}\{c(s,a) + J_k(f(s,a))\}$

Reinforcement Learning

Problems of Value Iteration:

for all $s \in \mathcal{S}$:$J_{k+1}(s) = \min_{a \in \mathcal{A}} \{c(s,a) + J_k(f(s,a))\}$

problems:

- Size of S (Chess, robotics, . . .) $\Rightarrow$ learning time, storage?
- 'model' (transition behaviour) $f(s,a)$ or $p(s', s, a)$ must be known!

Reinforcement Learning

Problems of Value Iteration:

for all $s \in \mathcal{S}: J_{k+1}(s) = \min_{a \in \mathcal{A}}\{c(s,a) + J_k(f(s,a))\}$

problems:

- Size of S (Chess, robotics, . . .) $\Rightarrow$ learning time, storage?
- 'model' (transition behaviour) $f(s,a)$ or $p(s', s, a)$ must be known!

Reinforcement Learning is dynamic programming for very large state spaces and/ or model-free tasks

Important contributions - Overview

- Real Time Dynamic Programming
  (Barto, Sutton, Watkins, 1989)

- Model-free learning (Q-Learning,(Watkins, 1989))

- neural representation of value function (or alternative function approximators)

Real Time Dynamic Programming (Barto, Sutton, Watkins, 1989)

Idea:

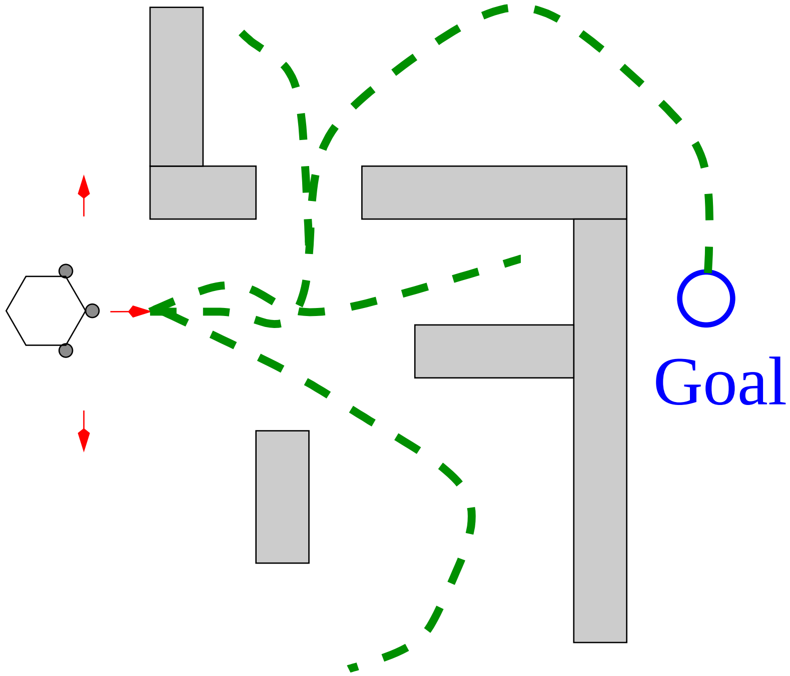
instead For all $s \in S$ now For some $s \in \mathcal{S}$ . . .

Real Time Dynamic Programming (Barto, Sutton, Watkins, 1989)

Idea:

instead For all $s \in S$ now For some $s \in \mathcal{S}$ . . .

$\Rightarrow$ learning based on trajectories (experiences)



Q-Learning

Idea (Watkins, Diss, 1989):
In every state store for every action the expected costs-to-go. $Q_\pi(s,a)$ denotes the expected future pathcosts for applying action a in state s (and continuing according to policy π):

Q-Learning

Idea (Watkins, Diss, 1989):
In every state store for every action the expected costs-to-go. $Q_\pi(s, a)$ denotes the expected future pathcosts for applying action a in state s (and continuing according to policy π):

$$Q_\pi(s, a) := (c(s, a) + J_\pi(s'))$$

Q-Learning

Idea (Watkins, Diss, 1989):
In every state store for every action the expected costs-to-go. $Q_\pi(s,a)$ denotes the expected future pathcosts for applying action a in state s (and continuing according to policy π):

$$Q_\pi(s,a) := \sum_{s' \in S} p(s', s, a)(c(s,a) + J_\pi(s'))$$

Q-Learning

Idea (Watkins, Diss, 1989):
In every state store for every action the expected costs-to-go. $Q_\pi(s,a)$ denotes the expected future pathcosts for applying action a in state s (and continuing according to policy π):

$$Q_\pi(s,a) := \sum_{s' \in S} p(s',s,a)(c(s,a) + J_\pi(s'))$$

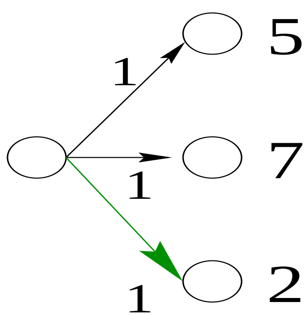
where $J_\pi(s')$ expected pathcosts when starting from s' and acting according to π

Q-learning: Action selection

is now possible without a model:

Original VI: state evaluation
Action selection:

$$\pi^*(s) \in \arg\min\{c(s,a) + J^*(f(s,a))\}$$

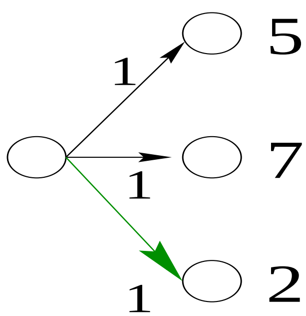


Q-learning: Action selection

is now possible without a model:

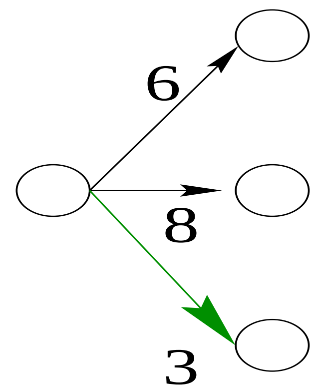
Original VI: state evaluation
Action selection:

$$\pi^*(s) \in \arg\min\{c(s,a) + J^*(f(s,a))\}$$



Q: state-action evaluation
Action selection:

$$\pi^*(s) = \arg\min Q^*(s,a)$$



Learning an optimal Q-Function

To find Q^*, a value iteration algorithm can be applied

$$Q_{k+1}(s, u) := (c(s, a) + J_k(s'))$$

Learning an optimal Q-Function

To find Q^*, a value iteration algorithm can be applied

$$Q_{k+1}(s,u) := \sum_{s' \in S} p(s', s, a)(c(s,a) + J_k(s'))$$

Learning an optimal Q-Function

To find Q^*, a value iteration algorithm can be applied

$$Q_{k+1}(s,u) := \sum_{s' \in S} p(s',s,a)(c(s,a) + J_k(s'))$$

where $J_k(s) = \min_{a' \in \mathcal{A}(s)} Q_k(s,a')$

Learning an optimal Q-Function

To find Q^*, a value iteration algorithm can be applied

$$Q_{k+1}(s,u) := \sum_{s' \in S} p(s', s, a)(c(s,a) + J_k(s'))$$

where $J_k(s) = \min_{a' \in \mathcal{A}(s)} Q_k(s, a')$

◇ Furthermore, learning a Q-function without a model, by experience of transition tuples $(s,a) \rightarrow s'$ only is possible:

Q-LEARNING (Q-Value Iteration + Robbins-Monro stochastic approximation)

$$Q_{k+1}(s,a) := (1-\alpha)\, Q_k(s,a) + \alpha\, (c(s,a) + \min_{a' \in \mathcal{A}(s')} Q_k(s', a'))$$

Summary Q-learning

Q-learning is a variant of value iteration when no model is available
it is based on two major ingredigents:

Summary Q-learning

Q-learning is a variant of value iteration when no model is available
it is based on two major ingredigents:

- uses a representation of costs-to-go for state/ action-pairs $Q(s,a)$

Summary Q-learning

Q-learning is a variant of value iteration when no model is available
it is based on two major ingredigents:

- uses a representation of costs-to-go for state/ action-pairs $Q(s, a)$
- uses a stochastic approximation scheme to incrementally compute expectation values on the basis of observed transititions $(s, a) \rightarrow s'$

Summary Q-learning

Q-learning is a variant of value iteration when no model is available
it is based on two major ingredigents:

- uses a representation of costs-to-go for state/ action-pairs $Q(s, a)$
- uses a stochastic approximation scheme to incrementally compute expectation values on the basis of observed transititions $(s, a) \rightarrow s'$

$\diamond$ converges under the same assumption as value iteration + *'every state/ action pair has to be visited infinitely often'* + conditions for stochastic approximation

Q-Learning algorithm

REPEAT

REPEAT

UNTIL

UNTIL

Q-Learning algorithm

REPEAT
  start in arbitrary initial state s_0; $t = 0$
  REPEAT

  UNTIL
UNTIL

Q-Learning algorithm

REPEAT

start in arbitrary initial state s_0; $t = 0$

REPEAT

choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$

UNTIL

UNTIL

Q-Learning algorithm

REPEAT

  start in arbitrary initial state s_0; $t = 0$

  REPEAT

    choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$

    or u_t according to an exploration scheme

  UNTIL

UNTIL

Q-Learning algorithm

REPEAT
  start in arbitrary initial state s_0; $t = 0$
  REPEAT
    choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$
    or u_t according to an exploration scheme
    apply u_t in the environment: $s_{t+1} = f(s_t, u_t, w_t)$

  UNTIL
UNTIL

Q-Learning algorithm

REPEAT

  start in arbitrary initial state s_0; $t = 0$

  REPEAT

    choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$

    or u_t according to an exploration scheme

    apply u_t in the environment: $s_{t+1} = f(s_t, u_t, w_t)$

    learn Q-value:

  UNTIL

UNTIL

Q-Learning algorithm

REPEAT
  start in arbitrary initial state s_0; $t = 0$
  REPEAT
    choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$
    or u_t according to an exploration scheme
    apply u_t in the environment: $s_{t+1} = f(s_t, u_t, w_t)$
    learn Q-value:
    $Q_{k+1}(s_t, u_t) := (1 - \alpha) Q_k(s_t, u_t) + \alpha(c(s_t, u_t) + J_k(s_{t+1}))$
    where $J_k(s_{t+1}) := \min_{a \in \mathcal{A}} Q_k(s_{t+1}, a)$
  UNTIL
UNTIL

Q-Learning algorithm

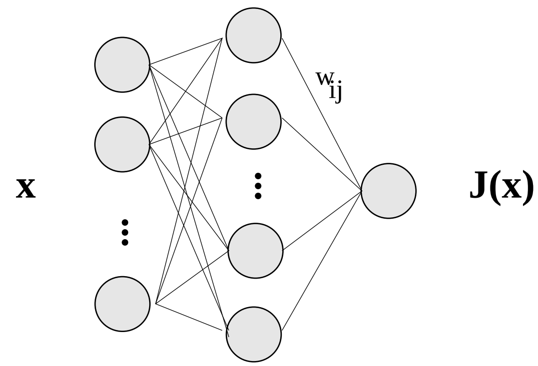
REPEAT
  start in arbitrary initial state s_0; $t = 0$
  REPEAT
    choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$
    or u_t according to an exploration scheme
    apply u_t in the environment: $s_{t+1} = f(s_t, u_t, w_t)$
    learn Q-value:
    $Q_{k+1}(s_t, u_t) := (1 - \alpha) Q_k(s_t, u_t) + \alpha(c(s_t, u_t) + J_k(s_{t+1}))$
    where $J_k(s_{t+1}) := \min_{a \in \mathcal{A}} Q_k(s_{t+1}, a)$
  UNTIL Terminal state reached
UNTIL

Q-Learning algorithm

REPEAT
  start in arbitrary initial state s_0; $t = 0$
  REPEAT
    choose action greedily $u_t := \arg\min_{a \in \mathcal{A}} Q_k(s_t, a)$
    or u_t according to an exploration scheme
    apply u_t in the environment: $s_{t+1} = f(s_t, u_t, w_t)$
    learn Q-value:
    $Q_{k+1}(s_t, u_t) := (1 - \alpha) Q_k(s_t, u_t) + \alpha(c(s_t, u_t) + J_k(s_{t+1}))$
    where $J_k(s_{t+1}) := \min_{a \in \mathcal{A}} Q_k(s_{t+1}, a)$
  UNTIL Terminal state reached
UNTIL policy is optimal ('enough')

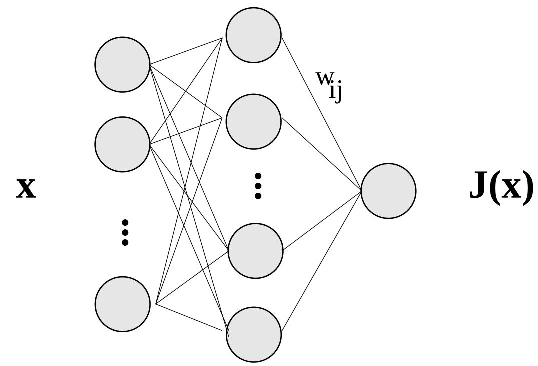
Representation of the path-costs in a function approximator

Idea: neural representation of value function (or alternative function approximators) (Neuro Dynamic Programming (Bertsekas, 1987))



Representation of the path-costs in a function approximator

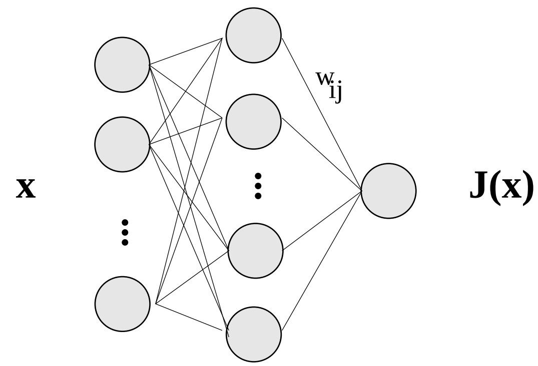
Idea: neural representation of value function (or alternative function approximators) (Neuro Dynamic Programming (Bertsekas, 1987))



⇒ few parameters (here: weights) specify value function for a large state space

Representation of the path-costs in a function approximator

Idea: neural representation of value function (or alternative function approximators) (Neuro Dynamic Programming (Bertsekas, 1987))

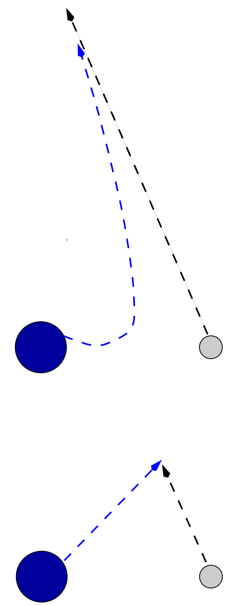


⇒ few parameters (here: weights) specify value function for a large state space

⇒ learning by gradient descent: $\frac{\partial E}{\partial w_{ij}} = \frac{\partial (J(s') - c(s,a) - J(s))^2}{\partial w_{ij}}$

Example: learning to intercept in robotic soccer

- as fast as possible (anticipation of intercept position)
- random noise in ball and player movement → need for corrections
- sequence of TURN(θ)and DASH(v)-commands required



⇒handcoding a routine is a lot of work, many parameters to tune!

Reinforcement learning of intercept

Goal: Ball is in kickrange of player

- state space: S^{work} = positions on pitch

Reinforcement learning of intercept

Goal: Ball is in kickrange of player

- state space: S^{work} = positions on pitch
- S^{+}: Ball in kickrange

Reinforcement learning of intercept

Goal: Ball is in kickrange of player

- state space: S^{work} = positions on pitch
- S^{+}: Ball in kickrange
- S^{-}: e.g. collision with opponent

Reinforcement learning of intercept

Goal: Ball is in kickrange of player

- state space: S^{work} = positions on pitch
- S^+: Ball in kickrange
- S^-: e.g. collision with opponent
- $c(s) = \begin{cases} 0 & , \quad s \in S^+ \\ 1 & , \quad s \in S^- \\ 0.01 & , \quad else \end{cases}$

Reinforcement learning of intercept

Goal: Ball is in kickrange of player

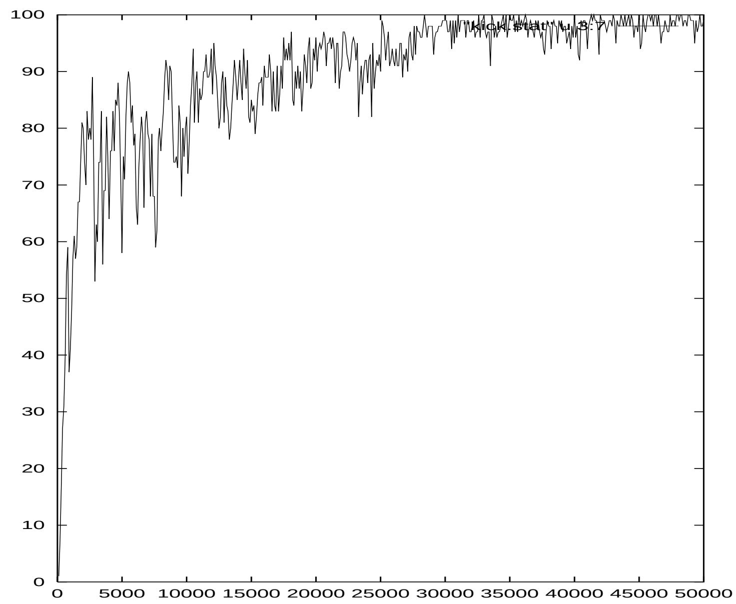
- state space: S^{work} = positions on pitch
- S^{+}: Ball in kickrange
- S^{-}: e.g. collision with opponent
- $c(s) = \begin{cases} 0 & , \quad s \in S^{+} \\ 1 & , \quad s \in S^{-} \\ 0.01 & , \quad else \end{cases}$
- Actions: $\textsc{turn}(10^o)$, $\textsc{turn}(20^o)$, ... $\textsc{turn}(360^o)$, ... $\textsc{dash}(10)$, $\textsc{dash}(20)$, ...

Reinforcement learning of intercept

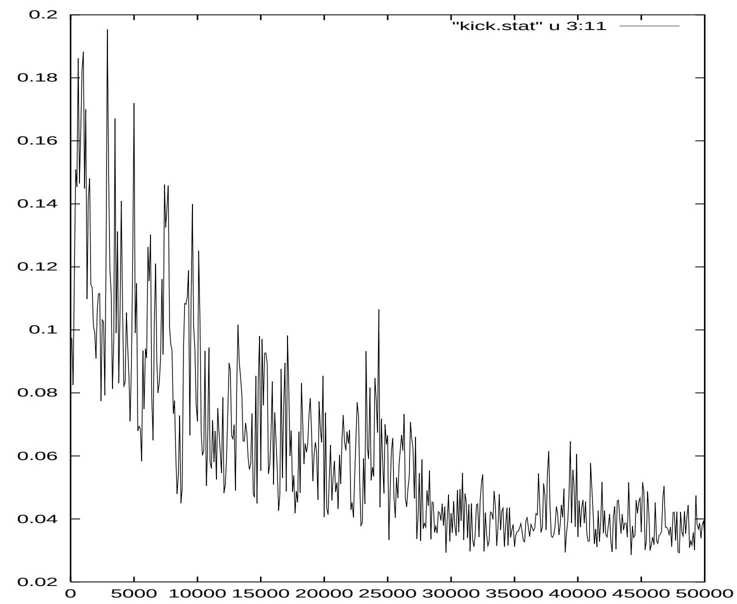
Goal: Ball is in kickrange of player

- state space: S^{work} = positions on pitch
- S^{+}: Ball in kickrange
- S^{-}: e.g. collision with opponent
- $c(s) = \begin{cases} 0 & , \quad s \in S^{+} \\ 1 & , \quad s \in S^{-} \\ 0.01 & , \quad else \end{cases}$
- Actions: $\textsc{turn}(10^o)$, $\textsc{turn}(20^o)$, ... $\textsc{turn}(360^o)$, ... $\textsc{dash}(10)$, $\textsc{dash}(20)$, ...
- neural value function (6-20-1-architecture)

Learning curves



Percentage of successes



Costs (time to intercept)